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**New York State Testing Program
Grade 7 Common Core
Mathematics Test**

Released Questions with Annotations

August 2013

Cassie rolls a fair number cube with 6 faces labeled 1 through 6. She rolls the number cube 300 times. Which result is **most** likely?

- A** Cassie will roll a 1 or a 2 about 50 times.
- B** Cassie will roll a 1 or a 2 exactly 50 times.
- C** Cassie will roll an even number about 150 times.
- D** Cassie will roll an even number exactly 150 times.

Key: C

Measured CCLS: 7.SP.6; 7.SP.7a

Commentary: The item measures 7.SP.7a, because it involves using a uniform probability model to determine probabilities of events. The item also measures 7.SP.6, because it requires predicting the approximate relative frequency given the probability.

Answer Choice A: "Cassie will roll a 1 or a 2 about 50 times." This response reflects the approximate number of times that any one side of the number cube will be rolled. The student may have found that the probability of rolling any one side was $\frac{1}{6}$, but did not include the probability of rolling the second side. A student who selects this response may not understand how to combine the probabilities of two events.

$$\frac{1}{6} + \frac{1}{6} = \frac{2}{6}$$

$$\frac{2}{6} \times 300 = 100$$

Answer Choice B: "Cassie will roll a 1 or a 2 exactly 50 times." This response reflects the approximate number of times that any one side of the number cube will be rolled. The student found that the probability of rolling any one side was $\frac{1}{6}$, but did not include the probability of rolling the second side. A student who selects this response may not understand how to combine the probabilities of two events. In addition, it is unlikely that experimental results will exactly match the prediction based on theoretical probability. It is more likely that a 1 will be rolled "about" 50 times, rather than "exactly" 50 times.

$$\frac{1}{6} + \frac{1}{6} = \frac{2}{6}$$

$$\frac{2}{6} \times 300 = 100$$

Answer Choice C: "Cassie will roll an even number about 150 times." The student correctly determined the approximate number of times the sides of the number cube labeled 2, 4, or 6 will be rolled. The student who selects this response likely used the probability that the number cube will land on a side labeled 2, 4, or 6 on any given roll to predict the total number of times that these events would occur out of 300 rolls.

$$\frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{3}{6}$$

$$\frac{3}{6} \times 300 = 150$$

Answer Choice D: "Cassie will roll an even number exactly 150 times." This response reflects the approximate number of times the sides of the number cube labeled 2, 4, or 6 will be rolled. The student likely found that the

probability of rolling one of these three sides was $\frac{3}{6}$, but did not allow that experimental results would not exactly match the prediction based on theoretical probability. It is more likely that an even number will be rolled "about" 150 times, rather than "exactly" 150 times. A student who selects this response may not understand the relationship between theoretical probability and experimental results.

Answer options A, B, and D are plausible but incorrect. They represent common student errors made when predicting the approximate relative frequency given the probability. Answer option C represents the approximate number of times the sides of the number cube labeled 2, 4, or 6 will be rolled out of 300 rolls.

What is the value of the expression below?

$$\frac{3}{8} + \left(-\frac{4}{5}\right) + \left(-\frac{3}{8}\right) + \frac{5}{4}$$

- A** 0
- B** $\frac{1}{20}$
- C** $\frac{9}{20}$
- D** $2\frac{4}{5}$

Key: C

Measured CCLS: 7.NS.1d

Commentary: The item measures 7.NS.1d because it assesses applying properties of operations as strategies to add and subtract rational numbers.

Answer Choice A: 0. This response may reflect a misunderstanding of additive inverses. The student may have recognized that $\frac{3}{8} + \left(-\frac{3}{8}\right) = 0$, but did not recognize that $-\frac{4}{5} + \frac{5}{4} = 0$ was a false statement.

$$\frac{3}{8} + \left(-\frac{4}{5}\right) + \left(-\frac{3}{8}\right) + \frac{5}{4} \rightarrow 0$$

Answer Choice B: $\frac{1}{20}$. This response may reflect a computational error when adding two fractions with unlike denominators. The student likely recognized that $\frac{3}{8} + \left(-\frac{3}{8}\right) = 0$, but incorrectly determined that $-\frac{4}{5} + \frac{5}{4} = \frac{1}{20}$.

$$\frac{3}{8} + \left(-\frac{4}{5}\right) + \left(-\frac{3}{8}\right) + \frac{5}{4} \rightarrow \frac{1}{20}$$

Answer Choice C: $\frac{9}{20}$. The student determined the correct value of the expression. Students who recognized the presence of additive inverses were rewarded with a simpler expression to calculate.

$$\frac{3}{8} + \left(-\frac{4}{5}\right) + \left(-\frac{3}{8}\right) + \frac{5}{4} = \frac{9}{20}$$

Answer Choice D: $2\frac{4}{5}$. This response reflects the value of the expression where the student disregarded all negative signs on the fractions, $\frac{3}{8} + \frac{4}{5} + \frac{3}{8} + \frac{5}{4} = 2\frac{4}{5}$. The student may not be correctly interpreting the notation of signs separated by parentheses in the expression. The student may only have a partial understanding of adding positive and negative fractions.

$$\frac{3}{8} + \left(-\frac{4}{5}\right) + \left(-\frac{3}{8}\right) + \frac{5}{4} \rightarrow 2\frac{4}{5}$$

Answer options A, B, and D are plausible but incorrect. They represent common student errors made when applying properties of operations as strategies to add and subtract rational numbers. Answer option C represents the correct process used to find the value of the given expression.

Carmine paid an electrician x dollars per hour for a 5-hour job plus \$70 for parts. The total charge was \$320. Which equation can be used to determine how much the electrician charged per hour?

- A** $5x = 320 + 70$
- B** $5x = 320 - 70$
- C** $(70 + 5)x = 320$
- D** $(70 - 5)x = 320$

Key: B

Measured CCLS: 7.EE.4a

Commentary: The item measures 7.EE.4a because it measures solving word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p , q , and r are specific rational numbers. The item assesses if students can determine the equation that can be used to solve the given word problem. Compare with the item on page 10, which also assesses 7.EE.4a.

Answer Choice A: $5x = 320 + 70$. This response is equivalent to $5x - 70 = 320$, which reflects the cost per hour of work, x , when the cost of the parts is subtracted from $5x$. The choice of this response indicates that the student may not understand that 70 is added to the hourly charge to determine the total of 320. A student who selects this response may have partial understanding of how to write equations in the form $px + q = r$, where p , q , and r are specific rational numbers, in the context of a word problem.

Answer Choice B: $5x = 320 - 70$. This response correctly shows that the cost of 5 hours of work is equal to the total cost excluding the cost associated with parts. This response may also be seen as equivalent to $5x + 70 = 320$, which reflects that the total cost of \$320 is the cost per hour, x , for 5 hours of work plus \$70 for parts. The student who selects this response determined an equation that can be used to solve the given word problem.

Answer Choice C: $(70 + 5)x = 320$. This response is equivalent to $75x = 320$, which reflects the cost per hour of work, x , for 75 hours of work with no parts included. A student who selects this response may have partial understanding of how the terms of an equation in the form of $px + q = r$, and $p(x + q) = r$ contribute to the value of r .

Answer Choice D: $(70 - 5)x = 320$. This response is equivalent to $65x = 320$, which reflects the cost per hour of work, x , for 65 hours of work with no parts included. A student who selects this response may have partial understanding of how the terms of an equation in the form of $px + q = r$, and $p(x + q) = r$ contribute to the value of r .

Answer options A, C, and D are plausible but incorrect. They represent common student errors made when determining an equation of the form $px + q = r$, and $p(x + q) = r$, where p , q , and r are specific rational numbers, that can be used to solve a word problem. Answer option B represents an equation in which \$320 is the cost per hour, x , for 5 hours of work plus \$70 for parts.

The relationship between the length of one side of a square, x , and the perimeter of the square, y , can be represented in an xy -plane by a straight line. Which of the points with coordinates (x, y) lie on the line?

- A** (2, 6)
- B** (2, 8)
- C** (6, 2)
- D** (8, 2)

Key: B

Measured CCLS: 7.RP.2d

Commentary: This item measures 7.RP.2d, because it assesses understanding of interpreting the meaning of point (x, y) on the graph of a proportional relationship. In this case, the proportional relationship is that the perimeter of a square is always four times the length of one of its sides.

Answer Choice A: (2, 6). This response reflects adding 4 to the x -coordinate instead of multiplying by 4. The student may have applied the 4 sides of the square to determine the y -coordinate, but chose an incorrect operation. A student who selects this response may have partial understanding of graphs of proportional relationships, but may not know how to apply a unit rate to determine an unknown point.

$$2 + 4 = 6 \rightarrow (2, 6)$$

Answer Choice B: (2, 8). The student determined a point that would lie on a graph where x represents the length of one side of a square and y represents the perimeter of the square. The student who selects this response may have determined that the unit rate of the graph would be 4 units of perimeter for each unit of side length, and then applied this unit rate to determine a point that would lie on the graph.

$$2 \times 4 = 8 \rightarrow (2, 8)$$

Students may have created a table of values and selected the correct solution choice by comparing them to their own table of values. Additionally, students may also have tested each coordinate according to the relationship above without directly calculating the unit rate (i.e., if the side length of the square is 2 the perimeter would be 8).

Answer Choice C: (6, 2). This response reflects adding 4 to the x -coordinate instead of multiplying by 4, and then reversing the order of the coordinates. The student may have applied the 4 sides of the square to determine the y -coordinate, but chose an incorrect operation, and then reversed the order of the coordinates. A student who selects this response may have partial understanding of graphs of proportional relationships, but may not know how to apply a unit rate to determine an unknown point, and may not understand the proportional relationship between the x and y on the coordinate plane.

$$2 + 4 = 6 \rightarrow (6, 2)$$

Answer Choice D: (8, 2). This response reflects a point that would lie on a graph, but with the order of the coordinates reversed. The student may have determined a point that would lie on a graph where x represents the perimeter of the square and y represents the length of one side of a square. The student who selects this response may have correctly determined that the unit rate of the graph would be 4 units of perimeter for each unit of side length and applied this unit rate to determine a point that would lie on the graph, but x and y were incorrectly defined.

$$2 \times 4 = 8 \rightarrow (8, 2)$$

Answer options A, C, and D are plausible but incorrect. They represent common student errors made when determining a point (x, y) on the graph of a proportional relationship in terms of a given situation. Answer option B represents determining that the unit rate of the graph would be 4 units of perimeter for each unit of side length, and then applying this unit rate to determine a point that would lie on the graph.

A crew of highway workers paved $\frac{2}{15}$ mile in 20 minutes. If they work at the same rate, what portion of a mile will they pave in one hour?

A $\frac{1}{150}$

B $\frac{2}{45}$

C $\frac{2}{5}$

D $\frac{5}{2}$

Key: C

Measured CCLS: 7.RP.1

Commentary: The item measures 7.RP.1 because it requires computing unit rates associated with ratios of fractions, including quantities measured in like or different units.

Answer Choice A: $\frac{1}{150}$. This response reflects the number of miles of road that workers pave in one minute. The student may have divided the distance by time, but did not convert the time in minutes to time in hours. A student who selects this response may be able to find unit rates, but was not precise in their use of units or in providing an answer in the units demanded by the question.

$$\frac{\frac{2}{15}}{20} = \frac{2}{15} \times \frac{1}{20} = \frac{1}{150}$$

Answer Choice B: $\frac{2}{45}$. This response reflects the number of miles of road that workers pave multiplied by time in hours. The student may have converted 20 minutes to $\frac{1}{3}$ hour, but multiplied, instead of dividing, the fractions to determine the unit rate. A student who selects this response may not understand how to use division of fractions to find unit rates associated with fractions.

$$\frac{2}{15} \times \frac{1}{3} = \frac{2}{45}$$

Answer Choice C: $\frac{2}{5}$. The student correctly determined the number of miles of road that workers pave in one hour. The student who selects this response computed the unit rate, in miles per hour, associated with paving $\frac{2}{15}$ miles in 20 minutes, which is equivalent to $\frac{1}{3}$ hour.

$$\frac{\frac{2}{15}}{\frac{1}{3}} = \frac{2}{15} \times 3 = \frac{2}{5}$$

Similarly, students may have used proportional reasoning to determine that if $\frac{2}{15}$ of mile can be paved in 20 minutes, then in one hour or 60 minutes three times as much road could be paved.

$$\frac{2}{15} \times 3 = \frac{6}{15} = \frac{2}{5}$$

Answer Choice D: $\frac{5}{2}$. This response reflects the number of hours it will take workers to pave one mile. The student may have converted 20 minutes to $\frac{1}{3}$ hour and used division of fractions to determine the unit rate, but reversed the position of the dividend and divisor in the equation. A student who selects this response may not understand how to create the expression needed to find the unit rate in miles per hour when the numerator and denominator are both fractions.

$$\frac{\frac{1}{3}}{\frac{2}{15}} = \frac{1}{3} \times \frac{15}{2} = \frac{5}{2}$$

Answer options A, B, and D are plausible but incorrect. They represent common student errors made when computing a unit rate associated with fractions. Answer option C represents the correct process used to compute the unit rate, in miles per hour, associated with paving $\frac{2}{15}$ miles in 20 minutes, which is equivalent to $\frac{1}{3}$ hour.

Which expression represents the sum of $(2x - 5y)$ and $(x + y)$?

- A** $3x - 4y$
- B** $3x - 6y$
- C** $x - 4y$
- D** $x - 6y$

Key: A

Measured CCLS: 7.EE.1

Commentary: The item measures 7.EE.1 because it involves the application of properties of operations as strategies to add, subtract, factor, and expand linear expressions. These expressions are considered linear because each term is either a constant or the product of a constant and the first power of a variable.

Answer Choice A: $3x - 4y$. The student determined the correct sum of the two given expressions. The student who selects this response has applied properties of operations to add two expressions and combine like terms. Two methods that could have been used are:

$$\begin{array}{l} \text{Method 1: } \begin{array}{l} (2x - 5y) + (x + y) \\ 2x + x + (-5y) + y \\ 3x + (-4y) \end{array} \\ \text{Method 2: } \begin{array}{l} (2x - 5y) + (x + y) \\ 2x + x - 5y + y \\ 3x - 4y \end{array} \end{array}$$

Answer Choice B: $3x - 6y$. This response reflects the sum of $(2x + x)$ and $-(5y + y)$. The student may have combined the coefficients of the y terms before applying the negative sign. A student who selects this response may not understand how to correctly apply the negative sign when rearranging expressions or combining terms with negative coefficients.

$$\begin{array}{l} (2x - 5y) + (x + y) \\ (2x + x) - (5y + y) \\ 3x - 6y \end{array}$$

Answer Choice C: $x - 4y$. This response reflects the sum of $(2x - 5y)$ and $(-x + y)$. The student likely attempted to add the original expressions but lacked precision. A student who selects this response may also not understand how to apply the negative sign in the first expression or correctly translate the language of the problem into a mathematical expression.

Answer Choice D: $x - 6y$. This response reflects the difference of the two given expressions. The student likely subtracted the expressions instead of adding. A student who selects this response may not understand how to correctly translate the language of the problem into a mathematical expression.

Answer options B, C, and D are plausible but incorrect. They represent common student errors made when applying properties of operations as strategies to add linear expressions with rational coefficients. Answer option A represents the correct process used to determine the sum of the two given expressions.

Which steps can be used to solve for the value of y ?

$$\frac{2}{3}(y + 57) = 178$$

- A** divide both sides by $\frac{2}{3}$, then subtract 57 from both sides
- B** subtract 57 from both sides, then divide both sides by $\frac{2}{3}$
- C** multiply both sides by $\frac{2}{3}$, then subtract 57 from both sides
- D** subtract $\frac{2}{3}$ from both sides, then subtract 57 from both sides

Key: A

Measured CCLS: 7.EE.4a

Commentary: The item measures 7.EE.4a because it involves solving equations of the form $p(x + q) = r$, where p , q , and r are specific rational numbers. This item specifically assesses if students can identify the sequence of operations required to solve equations in the form $p(x + q) = r$. While there are many different ways to solve an equation of this type, A is the only choice that leads to a solution in two steps. Compare with the item on page 4, which also assesses 7.EE.4a.

Answer Choice A: "divide both sides by $\frac{2}{3}$, then subtract 57 from both sides". The student determined the correct process needed to solve the given equation for the variable, y . The student who selects this response used the order of operations to determine that both sides of the equation may be divided by the coefficient and then the constant subtracted from both sides in order to solve the equation efficiently for the variable, y .

$$\begin{aligned} \frac{2}{3}(y + 57) &= 178 \\ \frac{\frac{2}{3}}{\frac{2}{3}}(y + 57) &= \frac{178}{\frac{2}{3}} \\ y + 57 &= 267 \\ y + 57 - 57 &= 267 - 57 \\ y &= 210 \end{aligned}$$

Answer Choice B: "subtract 57 from both sides, then divide both sides by $\frac{2}{3}$ ". This response does not lead to a solution in two steps; in fact, it reflects the opposite order of the usual solution method shown in A. A student who selects this response may not understand how to solve equations of the form $p(x + q) = r$ for the variable.

Answer Choice C: "multiply both sides by $\frac{2}{3}$, then subtract 57 from both sides." The student selected multiplication (instead of division as in the usual method shown in A) as the first step in the process to solve this equation for the variable, y . This response does not lead to a solution in two steps. A student who selects this response may not understand the inverse relationship between multiplication and division as they relate to solving equations for a variable.

Answer Choice D: "subtract $\frac{2}{3}$ from both sides, then subtract 57 from both sides." The student selected subtraction (instead of division as in the usual method shown in A) as the first step needed in the process to solve this equation for the variable. This response does not lead to a solution in two steps. A student who selects this response may not understand how to use inverse operations to solve equations of this form.

Answer options B, C, and D are all plausible but incorrect. They represent common student errors made when determining the process needed to solve an equation for the variable. Answer option A represents the correct process used to solve the given equation for the variable, y .

David bought a computer that was 20% off the regular price of \$1,080. If an 8% sales tax was added to the cost of the computer, what was the total price David paid for it?

- A** \$302.40
- B** \$864.00
- C** \$933.12
- D** \$1,382.40

Key: C

Measured CCLS: 7.EE.3

Commentary: The item measures 7.EE.3 because it measures solving a multi-step real-life problem posed with positive rational numbers in any form (whole numbers, fractions, and decimals). The solving process includes applying properties of operations to calculate with rational numbers and percents, and converting between forms as appropriate.

Answer Choice A: \$302.40. This response reflects 28% of the cost of the computer. The student may have multiplied the cost of the computer by the sum of the discount and tax. A student who selects this response may have a partial understanding of the relationship between discount and tax, and how these are related to the total cost of the computer.

$$1,080(0.20 + 0.08) = 302.40$$

Answer Choice B: \$864.00. This response reflects the cost of the computer after the discount. The student likely multiplied the cost of the computer by 0.8, but did not follow this step by applying the sales tax. A student who selects this response may not understand how a tax relates to the final cost of an item.

$$1,080 \times 0.8 = 864.00$$

Answer Choice C: \$933.12. The student determined the total amount paid for a computer, in dollars, including a discount and tax. The student who selects this response found the cost of the computer after the discount, and then the total cost after tax. Two methods that could have been used are:

<p>Method 1:</p> $1,080 \times 0.8 = 864.00$ $864.00 \times 1.08 = 933.12$	<p>Method 2:</p> $1,080 \times 0.2 = 216$ $1,080 - 216 = 864.00$ $864.00 \times 0.08 = 69.12$ $864.00 + 69.12 = 933.12$
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Answer Choice D: \$1,382.40. This response reflects 128% of the cost of the computer. The student multiplied the cost of the computer by the sum of the discount, tax, and one. A student who selects this response may have a partial understanding of the relationship between discount and tax, and how these are related to the total cost of the computer.

$$1,080(0.20 + 0.08 + 1) = 1,382.40$$

Answer options A, B, and D are plausible but incorrect. They represent common student errors made when solving multi-step real-life problems posed with positive rational numbers, represented by whole numbers and percents, using tools strategically. Answer option C represents the correct process used to find the total amount paid for a computer, in dollars, including a discount and tax.

Suzanne bought a sweater at the sale price of \$25. The original cost of the sweater was \$40. What percent represents the discount that Suzanne received when buying the sweater?

- A** 15%
- B** 37.5%
- C** 60%
- D** 62.5%

Key: B

Measured CCLS: 7.RP.3

Commentary: The item measures 7.RP.3 because it assesses the use of proportional relationships to solve multi-step ratio and percent problems.

Answer Choice A: 15%. This response reflects the difference between the original cost and the sale price. The student likely subtracted the sale price from the original cost to determine the difference in price, but did not follow this by dividing by 40 to determine the percent discount. A student who selects this response may not understand the relationship between a percent discount and the amount of a discount.

$$40 - 25 = 15$$

Answer Choice B: 37.5%. The student correctly determined the percent discount that was applied to a sweater when given the original cost and sale price. The student who selects this response computed the discount applied to the original cost of the sweater. Two methods that could have been used are:

$$\begin{array}{l} \text{Method 1:} \quad 40 - 25 = 15 \\ \quad \quad \quad (15 \div 40) \times 100 = 37.5 \end{array} \quad \begin{array}{l} 40x = 25 \\ \text{Method 2:} \quad x = \frac{25}{40} = 0.625 \\ \quad \quad \quad (1 - 0.625) \times 100 = 37.5 \end{array}$$

Answer Choice C: 60%. This response reflects the percent increase that can be applied to 25 to equal 40. The student may have subtracted the sale price from the original cost to determine the difference in price, but then divided by the sale price instead of the original cost. A student who selects this response may not understand the relationship between the percent discount and the original cost.

$$40 - 25 = 15$$

$$(15 \div 25) \times 100 = 60$$

Answer Choice D: 62.5%. This response reflects the percent of the original cost that is represented by the sale price. The student may have divided the sale price by the original cost to determine the percent of the original cost that is represented by the sale price, but did not follow this by subtracting this amount from one whole to determine the percent discount. A student who selects this response may not understand the relationship between the percentage of the original cost that is represented by the sale price and the percent discount.

$$\begin{array}{l} 40x = 25 \\ x = \frac{25}{40} = 0.625 \\ \frac{25}{40} \times 100 = 62.5 \end{array}$$

Answer options A, C, and D are plausible but incorrect. They represent common student errors made when using proportional relationships to solve multi-step ratio and percent problems. Answer option B represents the correct process used to determine the percent discount that was applied to a sweater when given the original cost and sale price.

Leo bought a used car for x dollars. One year later the value of the car was $0.88x$. Which expression is another way to describe the change in the value of the car?

- A** 0.12% decrease
- B** 0.88% decrease
- C** 12% decrease
- D** 88% decrease

Key: C

Measured CCLS: 7.EE.2

Commentary: The item measures 7.EE.2 because it requires students to rewrite an expression to shed light on how the quantities in the expression are related in a problem context. Students should recognize that $0.88x$ represents 88% of the original value; rewriting $0.88x$ as the equivalent $x - 0.12x$ or $(1 - 0.12)x$ reveals that the change in the value of the car could also be described as a 12% decrease in value.

Answer Choice A: 0.12% decrease. This response reflects $\frac{1}{100}$ of the percent decrease. The student may have rewritten $0.88x$ as the equivalent $x - 0.12x$ or $(1 - 0.12)x$ to reveal a decrease of 0.12 but did not multiply by 100 to convert to a percent. A student who selects this response may have partial understanding of how the quantities in the expression are related and how decimals relate to percents.

$$1 - 0.88 = 0.12$$

Answer Choice B: 0.88% decrease. This response reflects $\frac{1}{100}$ of the percent of x remaining after the decrease. The student found the percent of x remaining after the decrease as a decimal, but did not multiply by 100 to convert to a percent. A student who selects this response may have partial understanding of how the quantities in the expression are related and how decimals relate to percents.

$$0.88x \rightarrow 0.88$$

Answer Choice C: 12% decrease. The student determined the percent decrease. The student who selects this response understands how the quantities in the given expression are related. The student may have been likely able to rewrite $0.88x$ as the equivalent $x - 0.12x$ or $(1 - 0.12)x$ to reveal a decrease of 0.12 and also multiply by 100 to convert to a percent.

Answer Choice D: 88% decrease. This response reflects the percent of x remaining after the decrease. A student who selects this response may have partial understanding of how the quantities in the expression are related.

$$0.88x \rightarrow 0.88 \times 100 \rightarrow 88$$

Answer options A, B, and D are plausible but incorrect. They represent common student errors made when relating the quantities in the given expression. Answer option C represents the correct explanation of the meaning of 0.88 in the given expression.

Last week Len spent \$18 to bowl 4 games. This week he spent \$27 to bowl 6 games. Len owns his bowling ball and shoes, so he only has to pay for each game that he bowls. If each of these bowling games costs the same amount of money, what is the constant of proportionality between the money spent and the number of games played?

- A** 1.5
- B** 2.0
- C** 4.5
- D** 9.0

Key: C

Measured CCLS: 7.RP.2b

Commentary: The item measures 7.RP.2b because it requires the student to identify the constant of proportionality (unit rate) from a verbal description of a proportional relationship.

Answer Choice A: 1.5. This response reflects the ratio of the number of games played this week to the number of games played last week. The student may have divided the 6 games played this week by the 4 games played last week. A student who selects this response may not understand that the “constant of proportionality” being found in this problem relates the amount of money spent with the number of games played.

$$\frac{6}{4} = 1.5$$

Answer Choice B: 2.0. This response reflects the difference between the number of games played this week and the number of games played last week. The student may have subtracted the 4 games played last week from the 6 games played this week. A student who selects this response may not understand that the “constant of proportionality” being found in this problem relates the amount of money spent with the number of games played.

$$6 - 4 = 2$$

Answer Choice C: 4.5. The student correctly determined the constant of proportionality between the total money spent and the number of games played. The student understood the meaning of the phrase “constant of proportionality,” and simply found the unit rate (price per game).

$$\frac{18}{4} = 4.5 \text{ or } \frac{27}{6} = 4.5$$

Answer Choice D: 9.0. This response is twice the value of the constant of proportionality. The student may determine the unit rate for each week and then found the sum of these. A student who selects this response may not understand that the constant of proportionality is one value that describes the proportional relationship.

$$\frac{18}{4} = 4.5$$

$$\frac{27}{6} = 4.5$$

$$4.5 + 4.5 = 9$$

Answer options A, B, and D are plausible but incorrect. They represent common student errors made when identifying the constant of proportionality (unit rate) in verbal descriptions of proportional relationships. Answer option C represents a correct process used to identify the constant of proportionality (unit rate) in verbal descriptions of proportional relationships.

Julia's service charge at a beauty salon was \$72.60, before tax. The sales tax rate was 8%. If she added 20% of the amount before tax as a tip, how much did she pay for the service at the salon?

- A \$87.12
- B \$92.93
- C \$100.60
- D \$145.20

Key: B

Measured CCLS: 7.RP.3

Commentary: The item measures 7.RP.3, because it measures the use of proportional relationships to solve multi-step ratio and percent problems.

Answer Choice A: \$87.12. This response reflects the cost of the services plus 20%. The student likely increased the cost of the services by the 20% tip, but did not apply the 8% tax. A student who selects this response may not understand the relationship between a tax and the original amount.

$$72.60(1 + 0.2) = 87.12$$

Answer Choice B: \$92.93. The student determined the total amount of a service at a salon, in dollars, including tax and tip. The student who selects this response applied a 20% tip and 8% tax to the original cost. Two methods that could have been used are:

<p style="margin-left: 40px;">$72.60 \times 0.08 \approx 5.81$</p> <p>Method 1: $72.60 \times 0.2 = 14.52$</p> <p style="margin-left: 40px;">$72.60 + 5.81 + 14.52 = 92.93$</p>	<p style="margin-left: 40px;">$72.60 \times 1.08 \approx 78.41$</p> <p>Method 2: $78.41 + (72.60 \times 0.2) = 92.93$</p>
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Answer Choice C: \$100.60. This response reflects the cost of the services plus \$28. The student likely increased the cost of the services by the sum of a \$20 discount and an \$8 tax, with both values as a number of dollars instead of a percent of the original amount. A student who selects this response may not understand the relationship between a percent of the original amount and an amount in dollars.

$$72.60 + 28 = 100.60$$

Answer Choice D: \$145.20. This response reflects the cost of the services plus a 20% tip and 80% tax. The student likely increased the cost of the services by the 20% tip, but incorrectly converted the 8% tax to 0.8. A student who selects this response may not understand the relationship between a percent and the decimal representation of a percent or may lack precision when transforming from one form to another.

$$72.60(1 + 0.2) + 72.60(0.8) = 145.20$$

Answer options A, C, and D are plausible but incorrect. They represent common student errors made when using proportional relationships to solve multi-step ratio and percent problems. Answer option B represents the correct process used to determine the total amount of a service at a salon, in dollars, including tax and tip, where both the tax and tip are given as percents.

A group of friends went to lunch. The bill, before sales tax and tip, was \$37.50. A sales tax of 8% was added. The group then tipped 18% on the amount after the sales tax was added. What was the amount, in dollars, of the sales tax?

Show your work.

Answer \$ _____

What was the total amount the group paid, including tax and tip?

Show your work.

Answer \$ _____

Measured CCLS: 7.RP.3

Commentary: The item measures 7.RP.3 because it assesses using proportional relationships to solve multi-step ratio and percent problems.

Extended Rationale: This first question asks students to determine the tip amount, in dollars, and the second questions asks for the total amount of a restaurant bill, in dollars, including tax and tip. Students must include a set of computations that will lead to a correct response where work is provided to defend each step in the process. As indicated in the rubric, student responses will be rated on whether they show sufficient work to indicate a thorough understanding of using proportional relationships to solve multi-step ratio and percent problems. The determining factor in demonstrating a thorough understanding is using mathematically sound procedures to lead to a correct response.

The correct sales tax amount, in dollars, based on the amount of the restaurant bill may be arrived at by:

Sales tax amount, in dollars:

Method 1: $37.50(0.08) = 3.00$

Method 2: $\frac{x}{37.50} = \frac{8}{100}$, where x is the amount of sales tax

$$100x = 37.50(8)$$

$$x = \frac{37.50(8)}{100}$$

$$x = 3.00$$

Two possible methods for determining the correct total amount of the bill, in dollars, including tax and tip are:

Method 1: $37.50 + 3.00 = 40.50$
 $40.50(0.18) = 7.29$
 $40.50 + 7.29 = 47.79$

Method 2: $37.50 + 3.00 = 40.50$
 $40.50(1 + 0.18) = 47.79$

SAMPLE STUDENT RESPONSES AND SCORES APPEAR ON THE FOLLOWING PAGES: