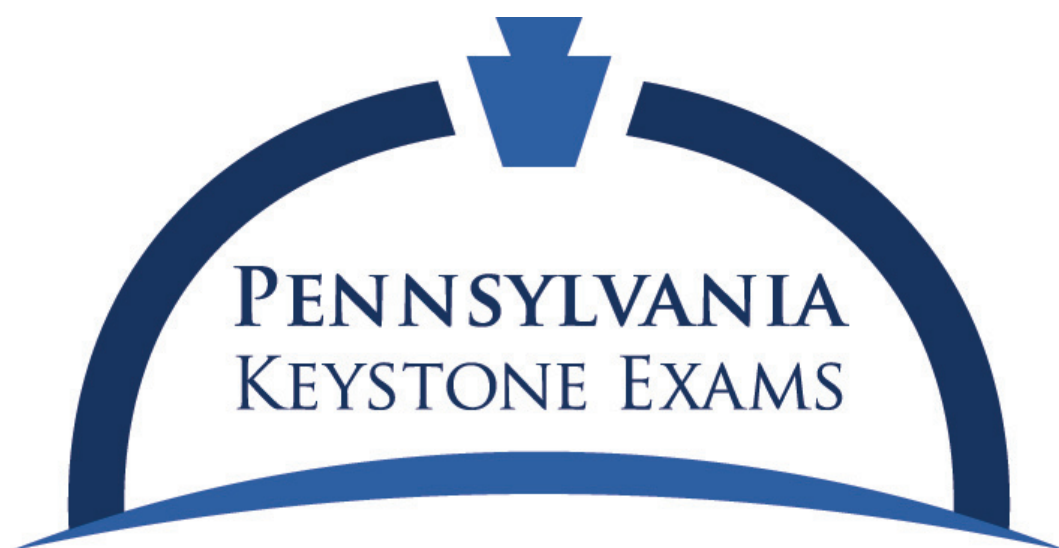




**pennsylvania**  
DEPARTMENT OF EDUCATION



ALGEBRA I

ITEM AND SCORING SAMPLER

2011

## MULTIPLE-CHOICE QUESTIONS

## A1.1.1.1.1

1. An expression is shown below.

$$2\sqrt{51x}$$

Which value of  $x$  makes the expression equivalent to  $10\sqrt{51}$ ?

- A. 5
- B. 25 \*
- C. 50
- D. 100

A student could determine the **correct** answer, option B, by factoring  $10\sqrt{51}$  as  $2 \times 5\sqrt{51}$ , then moving the 5 inside the radical as  $2\sqrt{51} \times 5 \times 5 = 2\sqrt{51} \times 25$ .

A student could arrive at an **incorrect** answer by either using an incorrect method or by making errors in computation. For example, a student would arrive at option A if he/she failed to square 5 when he/she moved it under the radical.

## A1.1.1.3.1

2. Simplify:

$$2(2\sqrt{4})^{-2}$$

A.  $\frac{1}{8}$  \*

B.  $\frac{1}{4}$

C. 16

D. 32

A student could determine the **correct** answer, option A, by recognizing

$$2(2\sqrt{4})^{-2} = \frac{2}{2\sqrt{4} \times 2\sqrt{4}} = \frac{2}{2 \times 2 \times \sqrt{4} \times \sqrt{4}} = \frac{1}{2 \times 4} = \frac{1}{8}.$$

A student could arrive at an **incorrect** answer by failing to follow correct order of operations or by not knowing how to use radicals or negative exponents. For example, a student would arrive at option D if he/she ignored the negative exponent and treated  $2(2\sqrt{4})^{-2}$  as  $2(2\sqrt{4})^2$ .

## A1.1.1.5.1

3. A polynomial expression is shown below.

$$(mx^3 + 3)(2x^2 + 5x + 2) - (8x^5 + 20x^4)$$

The expression is simplified to  $8x^3 + 6x^2 + 15x + 6$ . What is the value of  $m$ ?

- A. -8
- B. -4
- C. 4 \*
- D. 8

A student could determine the **correct** answer, option C, by using correct order of operations and the distributive property to expand  $(mx^3 + 3)(2x^2 + 5x + 2)$  to  $2mx^5 + 5mx^4 + 2mx^3 + 6x^2 + 15x + 6$ . The student could then combine like terms and realize that  $2mx^5 - 8x^5 = 0x^5$ , so  $2m = 8$  and  $m = 4$ .

A student could arrive at an **incorrect** answer by failing to follow order of operations, making an error with the distributive property, or incorrectly combining like terms. For example, a student would arrive at option D if he/she failed to distribute and then set  $mx^3 = 8x^3$ , so  $m = 8$ .

## A1.1.1.5.2

4. Which is a factor of the trinomial  $x^2 - 2x - 15$ ?

- A.  $(x - 13)$
- B.  $(x - 5)$  \*
- C.  $(x + 5)$
- D.  $(x + 13)$

A student could determine the **correct** answer, option B, by factoring the trinomial  $x^2 - 2x - 15$  as  $(x - 5)(x + 3)$  and identifying  $(x - 5)$  as a factor.

A student could arrive at an **incorrect** answer by failing to correctly factor the trinomial. For example, a student would arrive at option C if he/she factored  $x^2 - 2x - 15$  as  $(x + 5)(x - 3)$  and identified  $(x + 5)$  as a factor.

## A1.1.1.5.3

5. Simplify:

$$\frac{x^2 - 3x - 10}{x^2 + 6x + 8} ; x \neq -4, -2$$

A.  $-\frac{1}{2}x - \frac{5}{4}$

B.  $x^2 - \frac{1}{2}x - \frac{5}{4}$

C.  $\frac{x - 5}{x + 4}$  \*

D.  $\frac{x + 5}{x - 4}$

A student could determine the **correct** answer, option C, by factoring both the numerator and denominator, then reducing  $\frac{x^2 - 3x - 10}{x^2 + 6x + 8} = \frac{(x - 5)(x + 2)}{(x + 4)(x + 2)} = \frac{x - 5}{x + 4}$ .

A student could arrive at an **incorrect** answer by failing to factor the numerator and denominator or by incorrectly factoring the numerator and denominator. For example, a student would arrive at option D by factoring  $\frac{x^2 - 3x - 10}{x^2 + 6x + 8}$  as  $\frac{(x + 5)(x - 2)}{(x - 4)(x - 2)}$ .

## A1.1.2.2.1

6. Anna burned 15 calories per minute running for  $x$  minutes and 10 calories per minute hiking for  $y$  minutes. She spent a total of 60 minutes running and hiking and burned 700 calories. The system of equations shown below can be used to determine how much time Anna spent on each exercise.

$$15x + 10y = 700$$

$$x + y = 60$$

What is the value of  $x$ , the minutes Anna spent running?

- A. 10
- B. 20 \*
- C. 30
- D. 40

A student could determine the **correct** answer, option B, by solving the system of equations using substitution. Solving the equation  $x + y = 60$  for  $y$  yields  $y = 60 - x$ . Substituting  $60 - x$  in the place of  $y$  in the equation  $15x + 10y = 700$  yields  $15x + 10(60 - x) = 700$ . Using the distributive property yields  $15x + 600 - 10x = 700$ . Combining like terms and subtracting 600 from both sides yields  $5x = 100$ . Dividing both sides by 5 yields  $x = 20$ .

A student could arrive at an **incorrect** answer by either using an incorrect method for solving a system of equations or by making errors in computation. For example, a student would arrive at option D by incorrectly solving for  $y$  as  $y = x + 60$  and then failing to distribute when substituting, yielding  $15x + x + 60 = 700$ . Combining like terms and subtracting 60 from both sides yields  $16x = 640$ . Dividing both sides by 16 yields  $x = 40$ .

## A1.1.2.2.2

7. Samantha and Maria purchased flowers. Samantha purchased 5 roses for  $x$  dollars each and 4 daisies for  $y$  dollars each and spent \$32 on the flowers. Maria purchased 1 rose for  $x$  dollars and 6 daisies for  $y$  dollars each and spent \$22. The system of equations shown below represents this situation.

$$5x + 4y = 32$$

$$x + 6y = 22$$

Which statement is true?

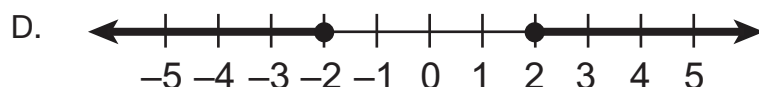
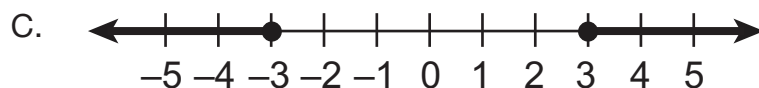
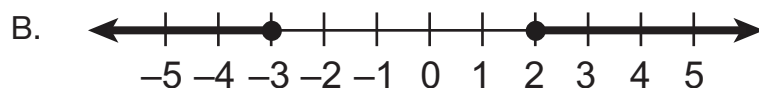
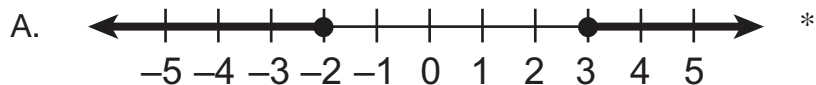
- A. A rose costs \$1 more than a daisy. \*
- B. Samantha spent \$4 on each daisy.
- C. Samantha spent more on daisies than she did on roses.
- D. Maria spent 6 times as much on daisies as she did on roses.

A student could determine the **correct** answer, option A, by solving the system of equations and correctly interpreting the solution  $x = 4$  and  $y = 3$ . The  $x$ -variable refers to the price of a rose and the  $y$ -variable refers to the price of a daisy.  $4 - 3 = 1$

A student could arrive at an **incorrect** answer by either making errors in solving the system of equations or by incorrectly interpreting the solution set. For example, a student would arrive at option B if he/she interpreted the  $x$ -value as the price of a daisy.

## A1.1.3.1.1

8. Which is a graph of the solution of the inequality  $|2x - 1| \geq 5$ ?



A student could determine the **correct** answer, option A, by simplifying the absolute value inequality.  $|2x - 1| \geq 5$  is equivalent to  $2x - 1 \geq 5$  and  $2x - 1 \leq -5$ . Solving the first inequality yields  $x \geq 3$ . Solving the second inequality yields  $x \leq -2$ .

A student could arrive at an **incorrect** answer by failing to split the absolute value inequality into simple inequalities before manipulating to solve the equation. For example, a student would arrive at option C if he/she first added 1 to each side of the absolute value inequality, divided both sides by 2, then split the absolute value inequality into simple inequalities.



## A1.1.3.1.3

9. A baseball team had \$1,000 to spend on supplies. The team spent \$185 on a new bat. New baseballs cost \$4 each. The inequality  $185 + 4b \leq 1,000$  can be used to determine the number of new baseballs ( $b$ ) that the team can purchase. Which statement about the number of new baseballs that can be purchased is true?
- A. The team can purchase 204 new baseballs.
  - B. The minimum number of new baseballs that can be purchased is 185.
  - C. The maximum number of new baseballs that can be purchased is 185.
  - D. The team can purchase 185 new baseballs, but this number is neither the maximum nor the minimum. \*

A student could determine the **correct** answer, option D, by solving the inequality and interpreting the solution  $b \leq 203.75$ . The variable  $b$  represents the number of baseballs that can be purchased. It is a true statement that  $185 \leq 203.75$ .

A student could arrive at an **incorrect** answer by either making errors in solving the system of equations or by incorrectly interpreting the solution set. For example, a student would arrive at option A if he/she switched the sign of the inequality when dividing by 4.

## A1.1.3.2.2

10. Tyreke always leaves a tip of between 8% and 20% for the server when he pays for his dinner. This can be represented by the system of inequalities shown below, where  $y$  is the amount of tip and  $x$  is the cost of dinner.

$$y > 0.08x$$

$$y < 0.2x$$

Which of the following is a true statement?

- A. When the cost of dinner,  $x$ , is \$10 the amount of tip,  $y$ , must be between \$2 and \$8.
- B. When the cost of dinner,  $x$ , is \$15 the amount of tip,  $y$ , must be between \$1.20 and \$3.00. \*
- C. When the tip,  $y$ , is \$3, the cost of dinner,  $x$ , must be between \$11 and \$23.
- D. When the tip,  $y$ , is \$2.40, the cost of dinner,  $x$ , must be between \$3 and \$6.

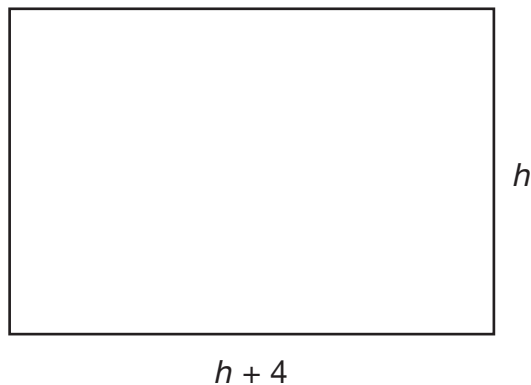
A student could determine the **correct** answer, option B, by interpreting the system of inequalities in the context of the problem situation. When 15 is substituted for the  $x$ -variable,  $y > 0.08(15)$  or  $y > 1.2$  and  $y < 0.2(15)$  or  $y < 3$ .

A student could arrive at an **incorrect** answer by either making errors in computation or in interpretation of the system of inequalities. For example, a student would arrive at option A if he/she incorrectly calculated  $0.08(10)$  as 8 and switched the signs of both inequalities.

## CONSTRUCTED-RESPONSE QUESTIONS

## A1.1.1

11. Keng creates a painting on a rectangular canvas with a width that is four inches longer than the height, as shown in the diagram below.



- A. Write a polynomial expression, in simplified form, that represents the area of the canvas.

Keng adds a 3-inch-wide frame around all sides of his canvas.

- B. Write a polynomial expression, in simplified form, that represents the **total area** of the canvas and the frame.

**Go to the next page to finish question 11.**

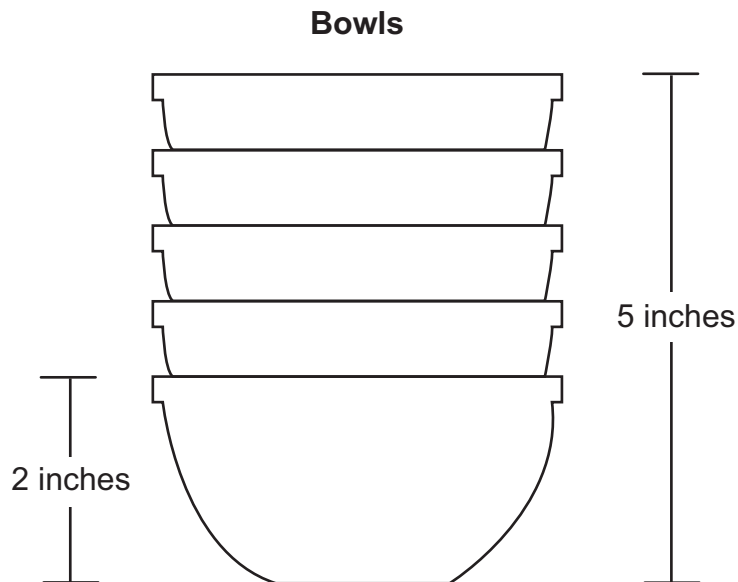
11. **Continued.** Please refer to the previous page for task explanation.

Keng is unhappy with his 3-inch-wide frame, so he decides to put a frame with a different width around his canvas. The total area of the canvas and the new frame is given by the polynomial  $h^2 + 8h + 12$ , where  $h$  represents the height of the canvas.

- C.** Determine the width of the new frame. Show all your work. Explain why you did each step.

## A1.1.2

12. The diagram below shows 5 identical bowls stacked one inside the other.



The height of 1 bowl is 2 inches. The height of a stack of 5 bowls is 5 inches.

- A.** Write an equation using  $x$  and  $y$  to find the height of a stack of bowls based on any number of bowls.

equation: \_\_\_\_\_

- B.** Describe what the  $x$  and  $y$  variables represent.

$x$ -variable: \_\_\_\_\_

$y$ -variable: \_\_\_\_\_

**Go to the next page to finish question 12.**

12. **Continued.** Please refer to the previous page for task explanation.

**C.** What is the height, in inches, of a stack of 10 bowls?

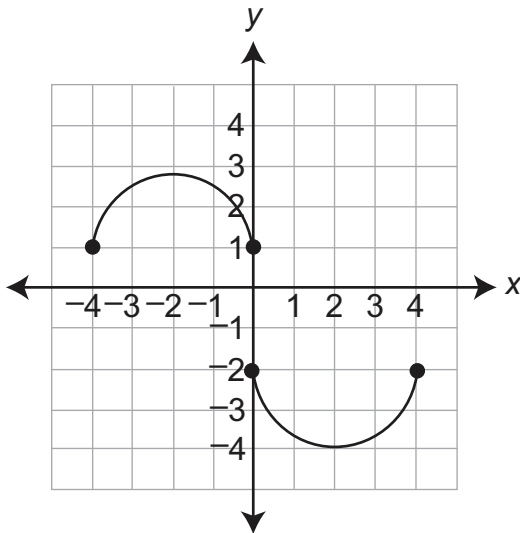
height: \_\_\_\_\_ inches

## MULTIPLE-CHOICE QUESTIONS

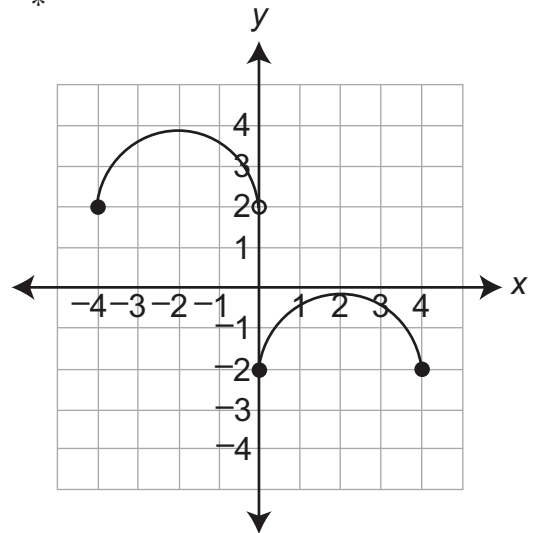
## A1.2.1.1.2

1. Which graph shows  $y$  as a function of  $x$ ?

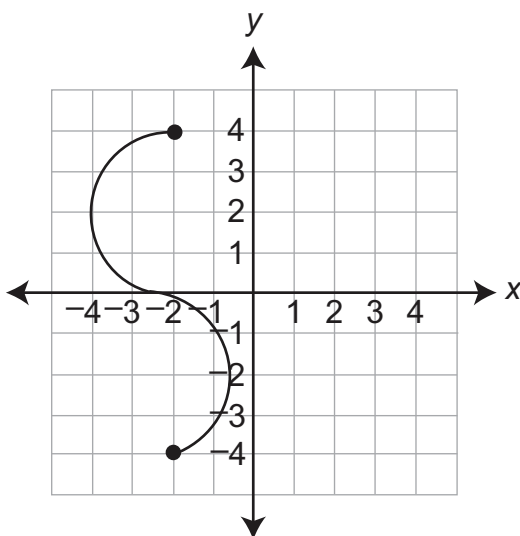
A.



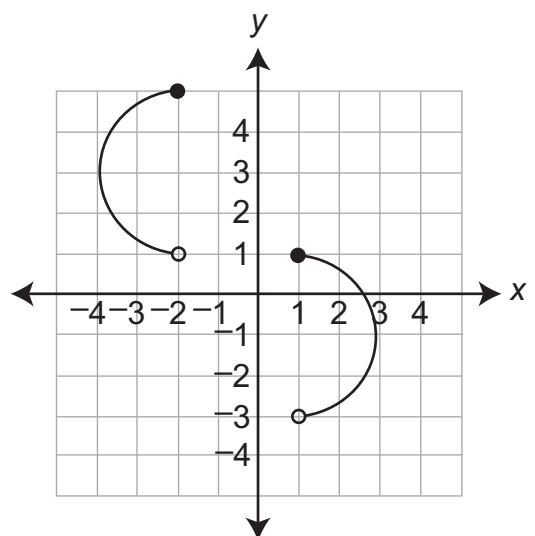
B. \*



C.



D.

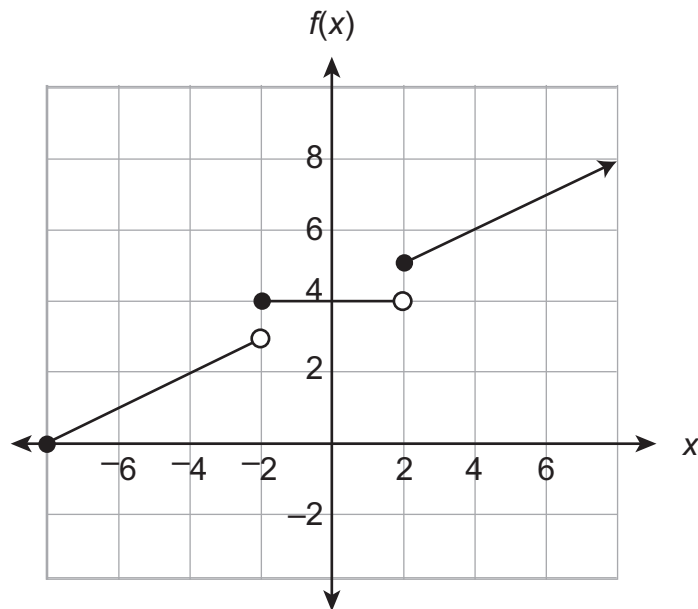


A student could determine the **correct** answer, option B, by examining the graphs of each of the relations and using the vertical line test.

A student could arrive at an **incorrect** answer by either not knowing the definition of a function or not knowing how to interpret closed and open circles. For example, a student would arrive at option C if he/she thought “function” meant “continuous”.

## A1.2.1.1.3

2. The graph of a function is shown below.



Which value is **not** in the range of the function?

- A. 0
- B. 3 \*
- C. 4
- D. 5

A student could determine the **correct** answer, option B, by examining the graph and seeing that it never intersects with the horizontal line  $y = 3$ .

A student could arrive at an **incorrect** answer by either not knowing the definition of range or not knowing how to interpret closed and open circles. For example, a student would arrive at option C if he/she thought range meant that there was only one value of  $x$  for each value of  $y$ .



## A1.2.1.2.1

3. A pizza restaurant charges for pizzas and adds a delivery fee. The cost ( $c$ ), in dollars, to have any number of pizzas ( $p$ ) delivered to a home is described by the function  $c = 8p + 3$ . Which statement is true?
- A. The cost of 8 pizzas is \$11.
  - B. The cost of 3 pizzas is \$14.
  - C. Each pizza costs \$8 and the delivery fee is \$3. \*
  - D. Each pizza costs \$3 and the delivery fee is \$8.

A student could determine the **correct** answer, option C, by interpreting the linear equation  $c = 8p + 3$  in the context of the problem situation.

A student could arrive at an **incorrect** answer by misinterpreting the linear equation  $c = 8p + 3$  in the context of the problem situation. For example, a student would arrive at option D if he/she interpreted the cost of a pizza to be the  $y$ -intercept and the delivery fee to be the slope.

## A1.2.1.2.2

4. The table below shows values of  $y$  as a function of  $x$ .

$x$	$y$
2	10
6	25
14	55
26	100
34	130

Which linear equation best describes the relationship between  $x$  and  $y$ ?

- A.  $y = 2.5x + 5$
- B.  $y = 3.75x + 2.5$  \*
- C.  $y = 4x + 1$
- D.  $y = 5x$

A student could determine the **correct** answer, option B, by identifying the linear equation which will map every value of  $x$  in the table to the corresponding value of  $y$ .

A student could arrive at an **incorrect** answer by checking only one of the  $(x, y)$  coordinate pairs in the table. For example, a student could arrive at option A if he/she only checked to see that the equation worked when  $x = 2$  and  $y = 10$ .

## A1.2.2.1.1

5. Jeff's restaurant sells hamburgers. The amount charged for a hamburger,  $h$ , is based on the cost for a plain hamburger plus an additional charge for each topping,  $t$ , as shown in the equation below.

$$h = 0.60t + 5$$

What does the number 0.60 represent in the equation?

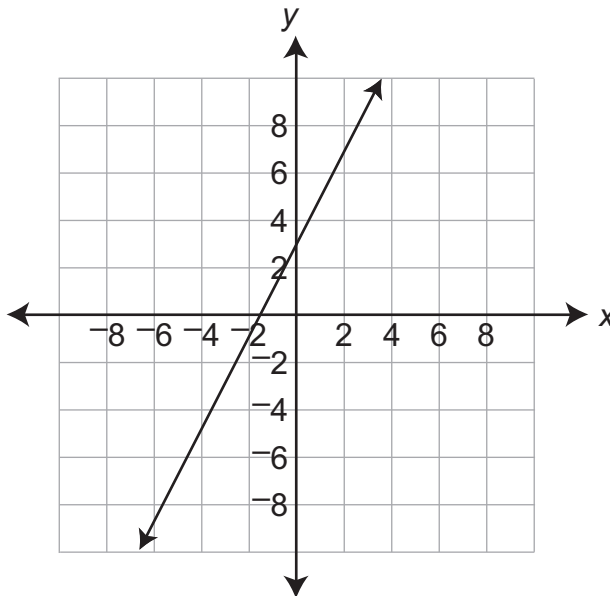
- A. the number of toppings
- B. the cost of a plain hamburger
- C. the additional cost for each topping \*
- D. the cost of a hamburger with 1 topping

A student could determine the **correct** answer, option C, by interpreting the linear equation  $h = 0.60t + 5$  in the context of the problem situation.

A student could arrive at an **incorrect** answer by misinterpreting the linear equation  $h = 0.60t + 5$  in the context of the problem situation. For example, a student would arrive at option A if he/she interpreted the number of toppings to be the rate of change.

## A1.2.2.1.3

6. A graph of a linear equation is shown below.



Which equation describes the graph?

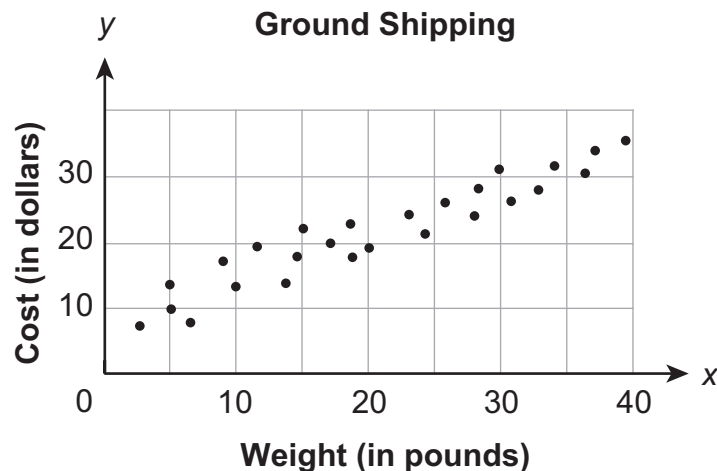
- A.  $y = 0.5x - 1.5$
- B.  $y = 0.5x + 3$
- C.  $y = 2x - 1.5$
- D.  $y = 2x + 3$  \*

A student could determine the **correct** answer, option D, by examining the graph to obtain the slope and y-intercept.

A student could arrive at an **incorrect** answer by either not knowing how to find the slope or y-intercept of a graph. For example, a student would arrive at option C if he/she used the x-intercept instead of the y-intercept.

## A1.2.2.2.1

7. The scatter plot below shows the cost,  $y$ , of ground shipping packages from Harrisburg, PA, to Minneapolis, MN, based on the package weight,  $x$ .



Which equation **best** describes the line of best fit?

- A.  $y = 0.37x + 1.57$
- B.  $y = 0.37x + 10.11$
- C.  $y = 0.68x + 2.32$
- D.  $y = 0.68x + 6.61$  \*

A student could determine the **correct** answer, option D, by drawing and deriving the equation of the line of best fit.

A student could arrive at an **incorrect** answer by either not knowing how to draw a line of best fit or not knowing how to find the equation of that line. For example, a student would arrive at option C if he/she drew a line such that all of the data points are at or above the line.

## A1.2.3.1.1

8. The daily high temperatures in degrees Fahrenheit in Allentown, PA, for a period of 10 days are shown below.

76 80 89 96 98 100 98 91 89 82

Which statement correctly describes the data?

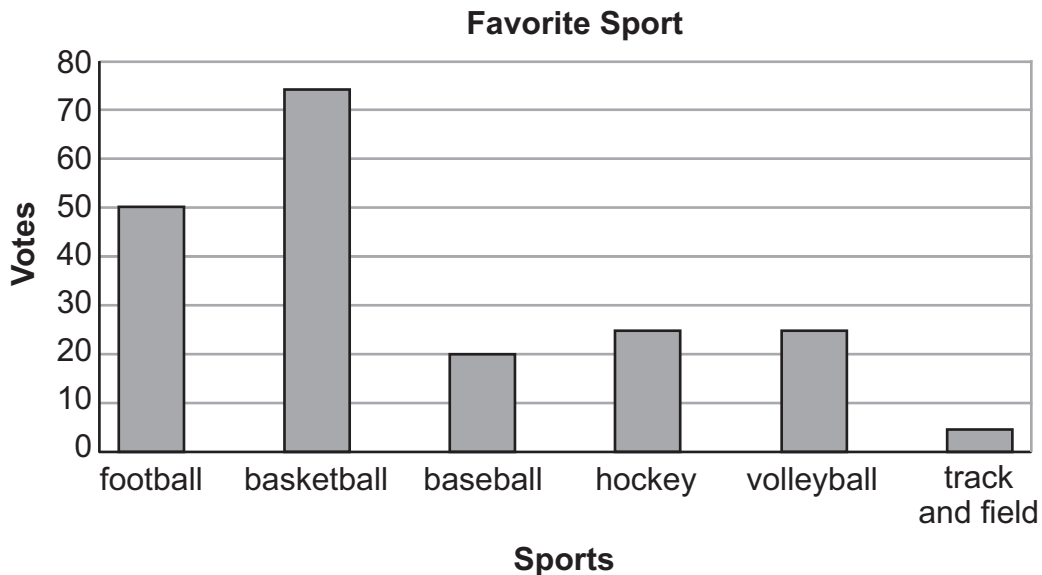
- A. The median value is 98.
- B. The interquartile range is 16. \*
- C. The lower quartile value is 76.
- D. The upper quartile value is 96.

A student could determine the **correct** answer, option B, by finding the difference between the third and first quartile. Arranging the data from lowest to highest shows that the median value is the average of 89 and 91. The third quartile value is the median of the upper half of the data, 98, and the first quartile value is the median of the lower half of the data, 82.  $98 - 82 = 16$ .

A student could arrive at an **incorrect** answer by not knowing how to use or compute median or interquartile range. For example, a student would arrive at option A if he/she confused median and mode.

## A1.2.3.2.1

9. Vy asked 200 students to select their favorite sport and then recorded the results in the bar graph below.



Vy will ask another 80 students to select their favorite sport. Based on the information in the bar graph, how many more students of the next 80 asked will select basketball rather than football as their favorite sport?

- A. 10 \*
- B. 20
- C. 25
- D. 30

A student could determine the **correct** answer, option A, by using the bar graph to obtain probabilities for basketball ( $75 \div 200 = 0.375$ ) and football ( $50 \div 200 = 0.25$ ), subtract the difference in the probabilities ( $0.375 - 0.25 = 0.125$ ) and multiply by the new sample ( $0.125 \times 80 = 10$ ).

A student could arrive at an **incorrect** answer by using an incorrect method or making a computational error. For example, a student would arrive at option C if he/she multiplied the probability difference by 200 instead of 80 ( $0.125 \times 200 = 25$ ).

## A1.2.3.3.1

10. A number cube with sides labeled 1–6 is rolled two times, and the sum of the numbers that end face up is calculated. What is the probability that the sum of the numbers is 3?

- A.  $\frac{1}{18}$  \*
- B.  $\frac{1}{12}$
- C.  $\frac{1}{9}$
- D.  $\frac{1}{2}$

A student could determine the **correct** answer, option A, by realizing that the possible combinations are 2 and 1 or 1 and 2. There are 2 ways to get a number for the first number cube out of 6 possible outcomes,  $\frac{2}{6}$ , and only 1 way to get a number for the second number cube,  $\frac{1}{6}$ . Multiplying the probabilities together  $\frac{2}{6} \times \frac{1}{6} = \frac{2}{36}$  which can be reduced to  $\frac{1}{18}$ .

A student could arrive at an **incorrect** answer by using an incorrect method or making a computational error. For example, a student would arrive at option C if he/she decided the probability for picking the first number cube was  $\frac{2}{6}$  and that the second number cube was also  $\frac{2}{6}$ , then  $\frac{2}{6} \times \frac{2}{6} = \frac{4}{36}$  which can be reduced to  $\frac{1}{9}$ .



## CONSTRUCTED-RESPONSE QUESTIONS

## A1.2.1

11. Hector's family is on a car trip.

When they are 84 miles from home, Hector begins recording their distance driven each hour in the table below.

Distance by Hour

Time in Hours	Distance in Miles
0	84
1	146
2	208
3	270

The pattern continues.

- A. Write an equation to find distance driven in miles ( $d$ ) after a given number of hours ( $h$ ).

- B. Hector also kept track of the remaining gasoline. The equation shown below can be used to find the gallons of gasoline remaining ( $g$ ) after distance driven ( $d$ ).

$$g = 16 - \frac{1}{20}d$$

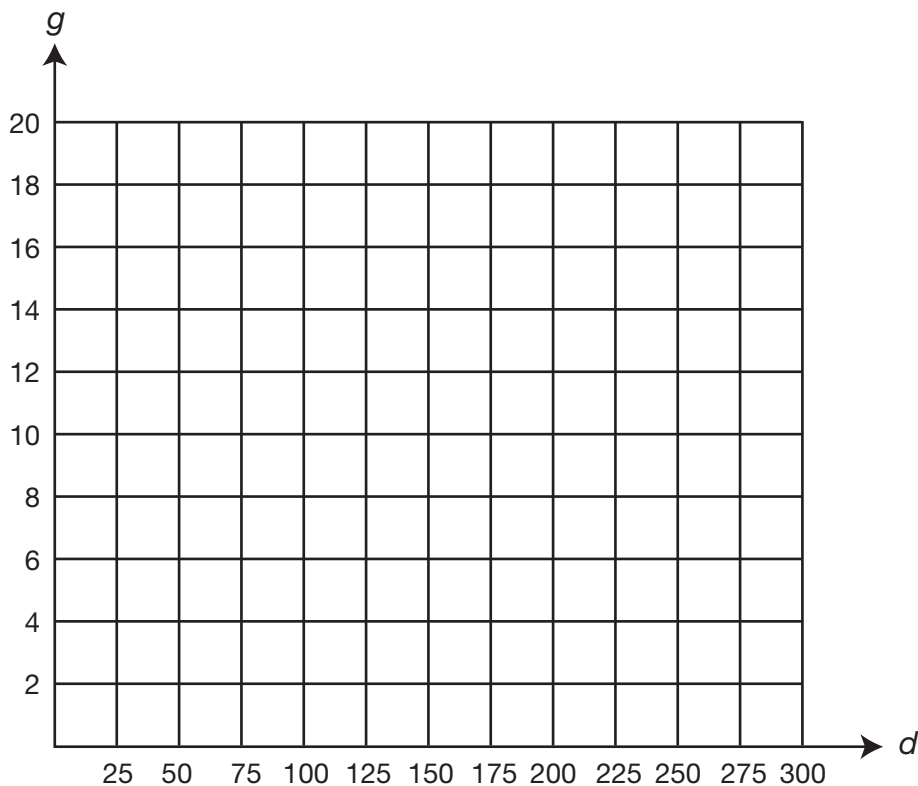
Use the equation to find the missing values for gallons of gasoline remaining.

Distance Driven in Miles ( $d$ )	Gallons of Gasoline Remaining ( $g$ )
100	
200	
300	

Go to the next page to finish question 11.

11. **Continued.** Please refer to the previous page for task explanation.

**C.** Draw the graph of the line formed by the points in the table from **part B**.



**D.** Explain why the slope of the line drawn in **part C** must be negative.

## A1.2.3

12. The weight (in pounds) of each wrestler on the high school wrestling team at the beginning of the season is listed below.

178 142 112 150 206 130

A. What is the median weight of the wrestlers?

median: \_\_\_\_\_ pounds

B. What is the mean weight of the wrestlers?

mean: \_\_\_\_\_ pounds

**Go to the next page to finish question 12.**

12. **Continued.** Please refer to the previous page for task explanation.

Two more wrestlers join the team during the season. The addition of these wrestlers has no effect on the mean weight of the wrestlers, but the median weight of the wrestlers increases 3 pounds.

- C.** Determine the weights of the two new wrestlers.

new wrestlers: \_\_\_\_\_ pounds and \_\_\_\_\_ pounds